Zero resistance at integer values of frustration in out-of-equilibrium classical Josephson-junction arrays

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The magnetic field dependence of the resistance of classical Josephson-junction arrays, with 500 000 or 1 000 000 junctions has been studied with significant transport currents through the array. The resistance vanishes into the noise only at integer values of the flux quantum per plaquette (integer frustration f) and is up to seven orders of magnitude larger than the noise floor at other fields. These arrays are apparently superconducting only at integer values of f. The only other surviving features of the complex modulation of the array resistance with field at low currents are small dips in the resistance vs field when $f=1/2$. The possible role of vortex pinning as being the source of these observations is discussed.

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Josephson-junction arrays[1] have served as model systems for a number of phenomena such as topological or Kosterlitz-Thouless-Berezinskii (KTB) phase transitions,[2] nonlinear dynamics,[3] frustration-related phenomena,[4] and vortex dynamics.[5] All of these are associated with classical behavior of the relative phases of the superconducting order parameters on each island of the array, even through the underlying physics is quantum mechanical. The relative phases behave quantum mechanically when the Josephson coupling energy, E_J, becomes less than the charging energy, E_C,[6] as long as $E_c \gg k_B T$. In this instance the ground state of an array can be tuned by varying a magnetic field perpendicular to the plane of the array, i.e., by varying the "frustration" or the magnetic flux in units of the flux quantum per plaquette,[7,8] altering the coupling to dissipation,[9] introducing disorder,[10] or changing the ratio E_J/E_C.[11] In the case of each of these continuous quantum phase transitions, the order-parameter phase difference between sites fluctuates while the order-parameter amplitude remains constant. Interestingly, one-dimensional (1D) metal wires and 1D arrays of junctions also exhibit what appear to be similar quantum phase transitions.[12] Even single junctions have been driven between insulating and superconducting behaviors by controlling the coupling to dissipative degrees of freedom.[13] Superconductivity is stabilized by dissipation, which locks the phase variables in the minima of a multidimensional free-energy landscape. Reducing the dissipation allows the phase to change by macroscopic quantum diffusion, which results in resistance. These dissipation driven transitions depend on ideas that go back to the pioneering work of Caldeira and Leggett,[14] and Schmid and Chakravarty.[15] Other types of quantum phase transitions have been modeled by variants of the Boson-Hubbard model.[16] This is closely related to the XY model but involves the addition of a capacitive energy to the Hamiltonian and the treatment of the phase and number of Cooper pairs on an island as quantum-mechanical conjugate variables.[11] This approach also appears to be relevant to the superfluid-Mott insulator transition of ultracold atoms in optical lattices.[17]

In this Brief Report we describe unusual behavior of arrays as a function of magnetic field. At relatively low temperatures, in the presence of a significant transport current through the array, but at levels short of driving the array normal, superconductivity (resistance in the noise) is found only at magnetic fields corresponding to integer values of frustration, f, with the array having a high resistance otherwise. This observation is in some sense the *opposite* of what was found in studies of the magnetic field dependence of the superconducting transition temperature of small cylinders (the Little-Parks experiment), where the transition temperature of the cylinder was driven to zero at half-integer values of the flux threading the cylinder[18] and remained nonzero at all other values of flux. The observation of superconductivity only at integer f resembles a series of phase transitions controlled by magnetic field. Although the measurements only extend down to a temperature of 1.9 K, the results are suggestive of some type of quantum phase transition, as the ground state of the array in the limit of zero temperature appears to be tuned by magnetic field.

The samples employed in these investigations were 2.0×2.0 cm^2 square arrays of either 1000×1000 or 500×1000 Nb-Nb junctions with Si-H alloy barriers, prepared some years ago by Sperry-Univac Corporation (now Unisys). The selective niobium anodization process was used to define the junction areas.[19] For electrical measurements on the array, the two opposite ends of the array were connected to a bar of Nb that provided pads for electrical contact. An optical microscope photograph of a small section of an array is shown in Fig. 1. Junctions in the arrays were approximately $6\ \mu m \times 6\ \mu m$ in area and the area of one of the square cells of the lattice was $28 \times 28\ \mu m^2$. The I-V characteristics of the single test junctions and the Fraunhofer pattern of the dependence of their critical currents on magnetic field were nearly ideal, suggesting sinusoidal current-phase relationships and homogeneity of the oxide barrier. The junction capacitance of similarly prepared junctions was measured to be $0.025\ pF/\mu m^2$ resulting in a single junction capacitance for these arrays of about 9×10^{-13} F.[20] We estimate the critical

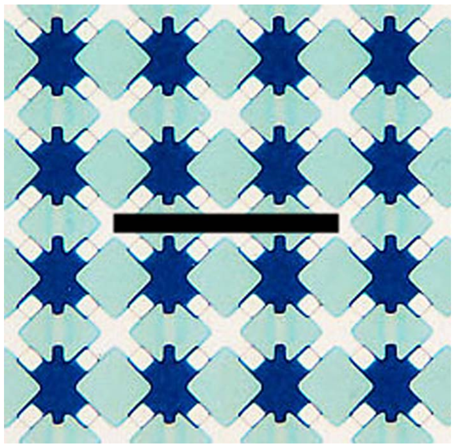

FIG. 1. (Color online) Optical microscope photograph of a small section of a typical array. The junctions are found at the overlap of the Nb electrodes which are crosses. The dark bar is 80 μm in length.

current of the individual junctions in the array to be about 1.65×10^{-7} A. How this is determined will be discussed below.

Measurements on the arrays were carried out in a liquid-helium cryostat that was equipped with an inner bath, which permitted temperature measurements down to about 1.9 K. The apparatus was suspended inside a high-permeability magnetic shield that reduced the ambient field to about 10^{-3} G, and magnetic fields for the measurements, transverse to the plane of the arrays were provided by a Helmholtz pair that produced a field of 45 G/A normal to the array with a variation of less than 5% over the 2.5×2.5 cm^2 area. The electrical leads were all twisted pairs and the apparatus itself was housed in a screened room with all electrical leads in and out of the room heavily filtered. Standard four-probe resistance measurement techniques were used, employing a Keithley 220 current source and a Keithley 182 nanovoltmeter.

It is important to realize that the behavior of Josephson-junction arrays is different from that of proximity-effect arrays in which superconducting disks are deposited on top of a normal-metal film. For proximity coupled arrays the transition of the *I*-*V* characteristic from normal state to superconducting behavior persists over an extend range of temperature. The proximity effect enlarges the effective radii of the superconducting disks while thermal fluctuations prevent the establishment of superconducting coupling between the disks. Eventually the order parameters of neighboring disks become strongly enough coupled to overcome thermal fluctuations and the entire structure behaves as a single film. In Josephson-junction arrays the intermediate stage is limited to about 0.1 K instead of several degrees. For the junctions considered here the resistance dropped from its normal-state value of 630 to 510 Ω.

A number of tests were carried out to determine the quality of the arrays. These included the demonstration of the KTB transition that is characterized by binding of vortex-antivortex pairs below a characteristic temperature, T_c. First, above T_c, the flux flow of free vortices causes a characteristic

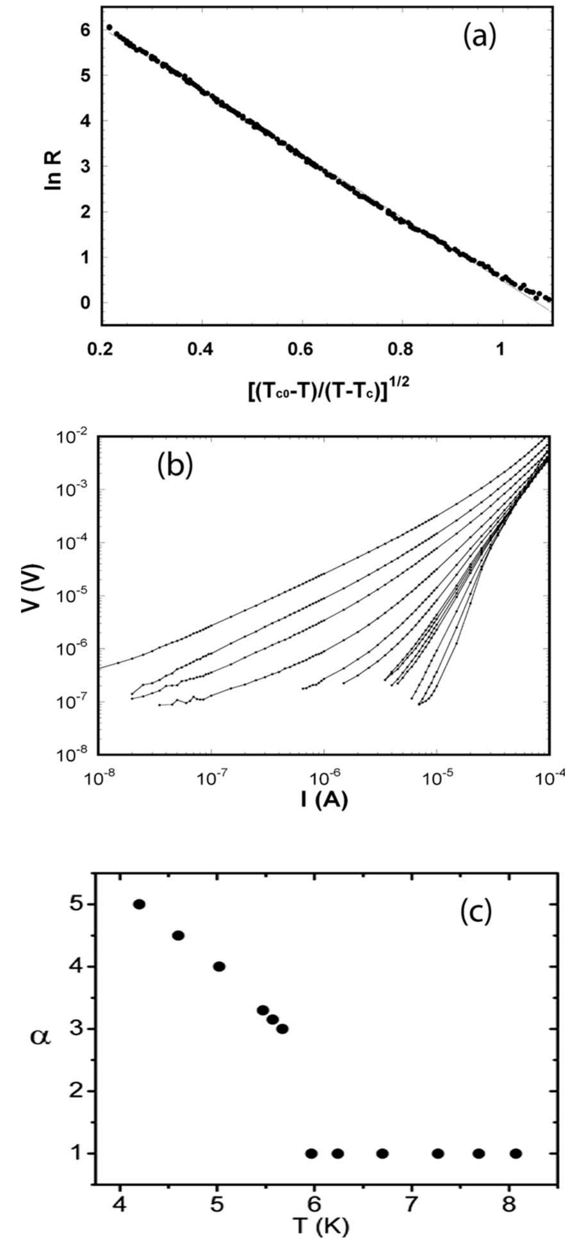

FIG. 2. (a) Fit of resistance versus temperature for a 500 000 junction array according to Eq. (1). Here, T_{c0} was taken to be 9.25 K and T_c was taken to be 5.85 K (which was determined from the jump in slope of ln *V* vs ln *I* of (c). (b) Nonlinear current-voltage characteristics in zero magnetic field for the temperatures 8.07 (top), 7.69, 7.27, 6.70, 6.24, 5.97, 5.67, 5.57, 5.47, 5.37, 5.02, 4.60, and 4.20 K (bottom). (c) Slope, α, of ln *V* vs ln *I* from (b) versus temperature with a distinct jump from 1 to 3 at 5.85 K.

temperature dependence of the resistance (measured in the zero-current limit). This dependence is given by

$$R = aR_N \exp\{-2[b(T_{c0}-T)/(T-T_c)]^{1/2}\}. \qquad (1)$$

Here, a and b are constants of order unity and T_{c0} is the mean-field transition temperature. The results of a fit of this function to the data are shown in Fig. 2(a).[21]

Above the KTB transition temperature, the flux flow of free vortices causes the *I*-*V* characteristic of the array to be linear. When the vortices bind into pairs of opposite helicity at and below T_c, the *I-V* characteristic of the array becomes nonlinear, of the form $V \sim I^{\alpha}$. A plot of ln V vs ln I yields $\alpha(T)$, which changes in a characteristic manner at the KTB temperature with the slope changing from 1 to 3 and continuing to increase with decreasing temperature. The parameter $\alpha(T)$ is related to the superfluid density and its abrupt change at the KTB transition temperature is indicative of the universal jump in the superfluid density at the transition. An expression for $\alpha(T)$ is given in Eq. (2),[21]

$$\alpha(T) = 1 + \pi K, \tag{2}$$

where K is a temperature-dependent parameter with limiting values of 0 when the vortex-antivortex interaction is fully screened and $2/\pi$ when interactions are strong. Plots of ln V vs ln I at different temperatures are shown in Fig. 2(b) and the variation in the variation of α with temperature is given in Fig. 2(c). These results confirm observations on these arrays reported many years ago.[22] They can also be used to estimate a lower bound on the critical current i_c of an individual junction in the array.[23] From the expression for the KTB transition temperature

$$k_B T_c = (\pi\hbar/4e) i_c(T_c) \tag{3}$$

and taking the KTB temperature T_c to be 5.85 K [from Fig. 2(c)], we find i_c to be 1.65×10^{-7} A as mentioned above. This is consistent with measurements of critical currents of single test junctions.

The second test, which is not shown, involved the study of the complex periodic variation in the array resistance with magnetic field. Many studies of classical arrays have been carried out to characterize this behavior.[24] The variation in resistance with frustration exhibits sharp minima at integer values of the frustration f, with secondary dips at various rational values, consistent with extensive past studies and with various simulations of frustrated XY models.[28]

We now consider the response of these arrays to perpendicular magnetic fields at high levels of transport current. One of the effects of high transport current is to greatly narrow the range over which the array exhibits zero resistance. A manifestation of this behavior is shown in Fig. 3, where the resistance of an array is plotted as a function of frustration f with a transport current of 120 μA flowing through the array at a temperature of 1.9 K. One notes that the resistance is effectively zero (in the noise at about 10^{-4} Ω) at integer values of f and is up to seven orders of magnitude higher at other values.

We now turn to a possible explanations of the behavior detailed in Fig. 3. One might argue that the very low (or zero) resistances at integer values of the flux quantum are the result of strong pinning of the vortex structure when the flux through each plaquette is either zero or an integer number of flux quanta and the absence of pinning at all other values of

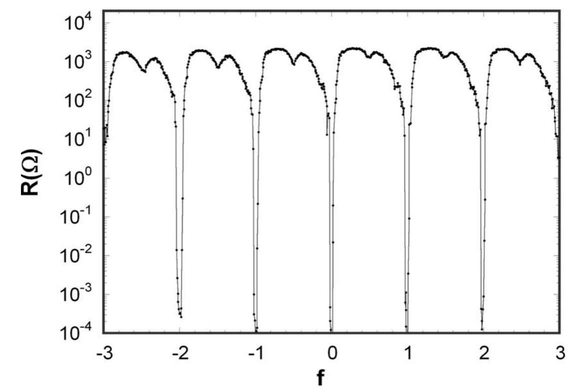

FIG. 3. Resistance R vs frustration f at 1.9 K with a measuring current of 120 μA. The resistance is effectively zero only at integer values of f. Although this is not shown, the transitions to zero resistance at integer values of f become sharper as the temperature is reduced.

flux per plaquette. An explanation involving pinning is very different from the physics of the linear regime at low currents in which a complex variation in resistance with field is the result of quantum interference effects. Pinning has been a central physical theme in the analysis the behavior of proximity coupled arrays, first considered some years ago by Rzchowski *et al.*[25] and more recently in the case of films with a lattice of holes by Montero *et al.*[26] and Raedts *et al.*[27] In each instance the modulation of the resistance going on and off integer values of frustration was far weaker than reported here. However pinning is a very unlikely explanation of the observations reported here for the following reason: from the critical current of a single junction, 1.65×10^{-7} A, computed from Eq. (3), the *maximum* single junction coupling energy $E_J = 5.3 \times 10^{-13}$ J. The barrier for vortex motion of a large array is about 0.2 of the coupling energy of an individual junction, making the barrier height smaller than $k_B T$ even at zero current.[23] Although we have no physical model explaining our observations, approaches to calculating the current-voltage characteristics of arrays in a dynamical regime[28] might be able to explain the data presented here.

The phase diagram of an array at high-current bias, in the space of field and temperature is unusual, in that superconductivity of the array is found only at integer values of frustration. Whether the transition from nonsuperconducting to superconducting behavior at or near integer values of f is some type of quantum phase transition, another explanation of these observations, would require further experimental and theoretical works beyond the scope of this Brief Report.

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